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Engineering Notes

Single-Stage-to-Orbit Versus Two-Stage-to-Orbit Airbreathing Systems

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Nomenclature

m_i = initial mass of the space transportation system or stage, lbm
m_p = orbital payload mass of the space transportation system, lbm
Z = ratio of the initial mass of an airbreathing single-stage-to-orbit system to that of an airbreathing two-stage-to-orbit system of equal technological capability
Z_e = ratio of the empty mass of an airbreathing single-stage-to-orbit system to that of an airbreathing two-stage-to-orbit system of equal technological capability
Γ = ratio of initial space transportation system mass to orbital payload mass, or initial mass ratio
π_e = ratio of empty mass to initial mass for a stage
π_f = ratio of fuel plus oxidizer mass to initial mass for a stage

Subscripts

SSTO = single-stage-to-orbit system
TSTO = two-stage-to-orbit system

I. Introduction

ONE of the most important decisions in the design of an orbital space transportation system (STS) is whether it should have a single stage or multiple stages. The debate often comes down to single-stage-to-orbit (SSTO) versus two-stage-to-orbit (TSTO) versions. The purpose of this paper is to provide a simplified approach to finding which option provides the least initial mass, which can be important for systems that have initial mass limitations, such as runway capability.

The figure of merit for comparison of airbreathing SSTO and TSTO systems employed in what follows is the ratio of the required initial (or launch) mass of the SSTO STS to that of the TSTO STS for a given mission (e.g., orbital payload mass, required energy, etc.). The analysis is based upon the material found in Chapter 3 of [1]. Since the results are presented in the form of algebraic equations, readers may insert their own empirical information or rearrange them for their own purposes. The results are presented entirely in terms of parameters for which the significance and magnitude are very familiar to designers: namely, the ratio of empty mass to initial mass for the airbreathing SSTO STS π_e and the ratio of fuel plus oxidizer mass to initial mass for the airbreathing SSTO STS π_f. This closed-form algebraic approach provides a framework for making high-level decisions about a complex topic.

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II. Analysis

The initial mass ratio of any SSTO STS is given by ([1], pg. 133)

$$\Gamma_{\mathrm{SSTO}} = \frac{m_{i,\mathrm{SSTO}}}{m_{p,\mathrm{SSTO}}} = \frac{1}{1 - \pi_f - \pi_e} \tag{1}$$

where the attainable values of π_e and π_f represent the technological capabilities for structural efficiency (i.e., empty mass fraction) and combined aerodynamic and propulsive efficiency (i.e., fuel plus oxidizer mass fraction), respectively, available to the designer for the given mission. This relationship is universal in the sense that it applies to any type of propulsion. Equation (1) may also be rearranged to permit the calculation of the orbital payload mass of the SSTO STS:

$$m_{p,\mathrm{SSTO}} = m_{i,\mathrm{SSTO}}(1 - \pi_f - \pi_e) \tag{2}$$

It can also be shown that the minimum value of the initial mass ratio of an orbital airbreathing TSTO STS is given by {[1], Eqs. (3-31), pg. 127, Eq. (3-35), page 129, and pages 137–141}

$$\Gamma_{\mathrm{TSTO}} = \frac{m_{i,\mathrm{TSTO}}}{m_{p,\mathrm{TSTO}}} = \left[\frac{1}{\sqrt{1 - \pi_f} - \pi_e}\right]^2 \tag{3}$$

provided that the technological capabilities π_e and π_f for both stages of the airbreathing TSTO STS are equal to those of the orbital airbreathing SSTO STS and that each stage of the TSTO STS delivers half of the mass specific orbital energy required by the mission to its final payload. These are reasonable assumptions, especially because the variation of Γ_{TSTO} with the mass specific energy split between the two stages is very gradual ([1], pg. 140).

Equation (3) may also be rearranged to permit the calculation of the orbital payload mass of the airbreathing TSTO STS:

$$m_{p,\mathrm{TSTO}} = m_{i,\mathrm{TSTO}}(\sqrt{1 - \pi_f} - \pi_e)^2 \tag{4}$$

Consequently, the ratio of the initial mass of an airbreathing SSTO STS to that of an airbreathing TSTO STS of equal technological capability is given by

$$Z = \frac{m_{i,\mathrm{SSTO}}}{m_{i,\mathrm{TSTO}}} = \frac{m_{i,\mathrm{SSTO}}/m_{p,\mathrm{SSTO}}}{m_{i,\mathrm{TSTO}}/m_{p,\mathrm{TSTO}}} = \frac{[\sqrt{1 - \pi_f} - \pi_e]^2}{[1 - \pi_f - \pi_e]} \tag{5}$$

because $m_{p,\mathrm{SSTO}} = m_{p,\mathrm{TSTO}}$ for any given mission. Z is the desired figure of merit for comparison of airbreathing SSTO and TSTO systems. When Z is less than one, the SSTO version is preferable. When Z exceeds one, the TSTO version is preferable.

III. Results

The central results of this analysis are presented graphically in Fig. 1. The most convenient approach is to solve Eq. (5) quadratically for π_f as a function of π_e for a fixed value of Z, whence

$$\pi_f = 1 - \pi_e^2\left[\frac{1 - \sqrt{1 + [(Z-1)(1 + Z/\pi_e)]}}{Z - 1}\right]^2, \qquad Z \neq 1 \tag{6}$$

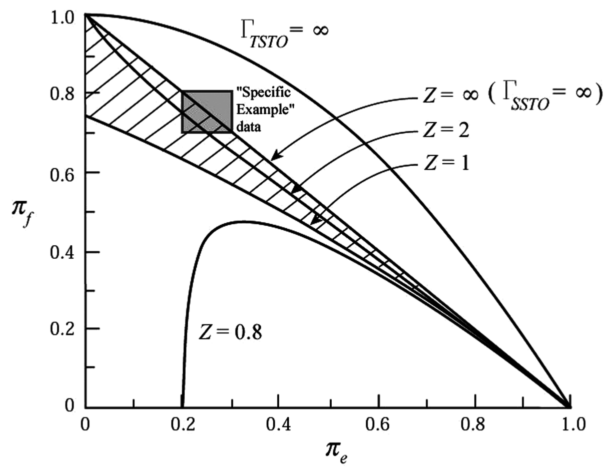

Fig. 1 Plot of π_f as a function of π_e for $Z = 0.80$ and 2.0 [Eq. (6)] $Z = 1$ [Eq. (7)], $\Gamma_{\text{SSTO}} = \infty$ [Eq. (1)] or $m_{p,\text{SSTO}} = 0$ [Eq. (2)] and $\Gamma_{\text{TSTO}} = \infty$ [Eq. (3)] or $m_{p,\text{TSTO}} = 0$ [Eq. (4)].

Since Eq. (6) is indeterminate for $Z = 1$, Eq. (5) is solved directly for $Z = 1$ to yield

$$\pi_f = 1 - \left(\frac{1 + \pi_e}{2}\right)^2 \tag{7}$$

Two other important cases are also shown in Fig. 1. First, according to Eq. (2), there can be no orbital payload mass for any SSTO STS when $\pi_f \geq 1 - \pi_e$; the equality corresponds to $\Gamma_{\text{SSTO}} = \infty$ [Eq. (1)] or $m_{p,\text{SSTO}} = 0$ [Eq. (2)]. Second, according to Eq. (4), there can be no orbital payload mass for any TSTO STS when $\pi_f \geq 1 - \pi_e^2$; the equality corresponds to $\Gamma_{\text{TSTO}} = \infty$ [Eq. (3)] or $m_{p,\text{TSTO}} = 0$ [Eq. (4)]. Regions of zero or negative m_p design space will be referred to as "forbidden" in what follows.

IV. Specific Example

During the National Aero-Space Plane (NASP) Program it was estimated that designers could reduce the ranges of π_e and π_f to $0.2 \leq \pi_e \leq 0.3$ and $0.7 \leq \pi_f \leq 0.8$, respectively ([1], pages 20–21).This region is shown as the shaded rectangle in Fig. 1. Since the NASP was intended to be an orbital airbreathing SSTO STS, the above analysis may be applied to this case. Comparing the shaded region with the remainder of Fig. 1, it may be concluded that half of the possible design points could be accomplished only by an airbreathing TSTO STS and the remaining half are squarely in the region where $Z \geq 1$, and the TSTO solution is preferred to the SSTO solution.

Equation (1) may be used to find that the lowest possible value of Γ_{SSTO} corresponding to the NASP design parameters is 10.0. Equation (3) may be used to find that the lowest possible value of Γ_{TSTO} corresponding to the NASP design parameters is 8.27. The ratio $Z = \Gamma_{\text{SSTO}}/\Gamma_{\text{TSTO}} = 10.0/8.27 = 1.21$ agrees with the results displayed in Fig. 1.

V. Empty Mass

When other figures of merit for comparison of space transportation systems are desired, they may generally be derived from the same material (see [1], page 138, for guidance). The total empty mass of the SSTO or TSTO STS is often considered to be the most important figure of merit for comparison purposes. In this case, the ratio of the total empty mass of the SSTO STS to that of the TSTO STS can be found, provided the stages have equal technological capability, to be

$$Z_e = \frac{m_{e,\text{SSTO}}}{m_{e,\text{TSTO}}} = \frac{m_{e,\text{SSTO}}/m_p}{m_{e,\text{TSTO}}/m_p} = \frac{Z}{(2 - \pi_e - \pi_f)} \tag{8}$$

For the results displayed in Fig. 1 corresponding to the conditions of Eq. (5), one can see by inspection that in the region of interest of Fig. 1 (i.e., either near the "Specific Example" box or between the lines $Z = 2$ and $Z = \infty$) that $1 - \pi_e - \pi_f \cong 0$, so that

$$Z_e \cong Z \tag{9}$$

Consequently, the two figures of merit yield similar conclusions for the airbreathing specific example of Fig. 1. Careful calculations lead to the same result.

VI. Conclusions

The results of calculations based on Eqs. (1–4), (6), and (7) are displayed together in Fig. 1. Proceeding from the upper right-hand corner to the lower left-hand corner of Fig. 1, the most important regions are described below.

The entire region above the line $\pi_f = 1 - \pi_e^2$ is forbidden. No successful SSTO or TSTO STS can be designed in this region.

The region between the line $\pi_f = 1 - \pi_e^2$ ($\Gamma_{\text{TSTO}} = \infty$) and the line $\pi_f = 1 - \pi_e$ ($Z = \infty$ or $\Gamma_{\text{SSTO}} = \infty$) is forbidden for SSTO systems. Only TSTO systems can be successfully designed in this region.

Any design point in the region above the line $\pi_f = 1 - \pi_e$ ($Z = \infty$ or $\Gamma_{\text{SSTO}} = \infty$) is forbidden for SSTO systems.

Any design point in the region below the line $\pi_f = 1 - \pi_e$ ($Z = \infty$ or $\Gamma_{\text{SSTO}} = \infty$) but above the line given by Eq. (7) for $Z = 1$ represents a STS for which the TSTO solution is preferred to the SSTO. This region is shown with cross-hatching in Fig. 1. For example, along the $Z = 2$ isoline shown in Fig. 1 the SSTO STS would have an initial mass double that of the TSTO STS for the same mission. It should be emphasized that it is possible to design SSTO systems in this region. Their TSTO counterparts simply have less initial mass and are therefore more desirable.

Any design point in the region below the line given by Eq. (7) for $Z = 1$ and above the π_e axis represents a STS for which the SSTO solution is preferred to the TSTO. For example, along the $Z = 0.8$ isoline shown in Fig. 1 the SSTO design would have an initial mass 80% as large as that of the TSTO design for the same mission. It should be emphasized that it is possible to design TSTO systems in this region. Their SSTO counterparts simply have less initial mass and are therefore more desirable.

The designer must reduce the values of π_e and π_f as much as possible in order to either reduce the initial mass of any STS or to make SSTO systems possible or competitive. The values of Z shown in Fig. 1 indicate the sensitivity of possible designs to changes of π_e and π_f. This sensitivity may also be explored by constructing and examining lines of constant Γ_{SSTO} [Eq. (1)] and Γ_{TSTO} [Eq. (3)] plotted for π_f as a function of π_e, as in Fig. 1. The intersections of $\Gamma_{\text{SSTO}} = \Gamma_{\text{TSTO}}$ in this representation will reproduce the line for $Z = 1$ given by Eq. (7).

Although this analysis applies only to systems with airbreathing propulsion and equal technological capability in every stage, similar analyses can be carried out for other situations. Readers are encouraged to pursue these options independently.

References

[1] Heiser, W. H., and Pratt, D. T., *Hypersonic Airbreathing Propulsion*, AIAA Education Series, AIAA, Washington, D.C., 1994.

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